THE MORDELL CONJECTURE 100 YEARS LATER
LIGHTNING TALKS – TITLES AND ABSTRACTS

All lightning talks are at most **five minutes** long. The schedule includes a two minute break to switch to the next speaker.

All lightning talks will take place in Room 10-250 at MIT pictured here `https://mordell.org/room.html`.

We refer to `www.mordell.org` for information on the conference.

Summary of the Schedule

Monday	Speaker
11:15 am	Marius Leonhardt
11:22 am	Florian Ito Sprung
11:29 am	Ashvin A. Swaminathan
11:36 am	Matt Broe
11:43 am	Aaron Landesman
11:50 am	Rina Paucar
11:57 am	Chris Xu

Monday	Speaker
3:10 pm	Ivan Aidun
3:17 pm	Lorenzo Furio
3:24 pm	Arav Karighattam
3:31 pm	Elvira Lupoian
3:38 pm	Boaz Moerman
3:45 pm	James Austin Myer
3:52 pm	Robin Visser

Tuesday	Speaker
11:15 am	Elyes Boughattas
11:22 am	Jerry Yu Fu
11:29 am	Asimina Hamakiotes
11:36 am	Maleeha Khawaja
11:43 am	Pietro Mercuri
11:50 am	Thibaut Misme
11:57 am	Madhavan Venkatesh

Date: July 6, 2024.

Tuesday	Speaker
3:10 pm	Niven Achenjang
3:17 pm	Luca Ferrigno
3:24 pm	Giorgio Navone
3:31 pm	Benjamin York
3:38 pm	Roy Zhao
3:45 pm	Sa'ar Zehavi
3:52 pm	Pratiksha Shingavekar

Session 1: Monday, July 8, 11:15 - 12:05

Monday	Speaker
11:15 am	Marius Leonhardt
11:22 am	Florian Ito Sprung
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Marius Leonhardt (Universität Heidelberg)

Affine Chabauty

Abstract: In joint work with M. Lütdke and J.S. Müller, we showed that the linear Chabauty–Kim locus of an affine hyperbolic curve is finite if the rank of its Jacobian is less than its genus plus a boundary contribution. I want to present this result and, if time permits, indicate work in progress that tries to turn it into an algorithm determining the Chabauty–Kim locus, and therefore the set of integral points, of the curve.

Florian Ito Sprung (Arizona State University)

Hilbert's 10th Problem for Number Fields

Abstract: We will illustrate a theorem of Poonen and Shlapentokh about stability of non-trivial Mordell–Weil ranks, and use it to prove the analogue of Hilbert's 10th Problem for rings of integers of some new number fields.

Ashvin A. Swaminathan (Harvard University)

A positive proportion of monic odd hyperelliptic curves have no unexpected quadratic points

Abstract: We prove that when monic odd hyperelliptic curves are ordered by the sizes of their coefficients, a positive proportion of them have no unexpected quadratic points — i.e., no points defined over quadratic fields except for those that arise by pulling back rational points from $\mathbb{P}^1$. To obtain this result, we develop and apply a generalization Selmer-group Chabauty (due to Poonen-Stoll). This is joint work with Jef Laga.

Matt Broe (Boston University)

The motive of the g-fold product of a CM elliptic curve

Abstract: Let k be a field, and E be a elliptic curve over k with complex multiplication. We construct a decomposition of the Chow motive of E^g which refines the Chow-Kunneth decomposition. Moonen asked whether each term in the refined decomposition has Chow groups concentrated in a single degree. We give an explicit counterexample over a number field, drawing on work of Bloch. When k is a global field of positive characteristic, we formulate a version of the Beilinson-Bloch conjecture for varieties over k, prove it in some special cases, and study its implications for Moonen's question over k.

Aaron Landesman (MIT)

The distribution of ranks of abelian varieties in quadratic twist families over function fields

Abstract: The minimalist conjecture predicts that, in quadratic twist families of abelian varieties, half have rank 0 and half have rank 1. This fits into the larger picture of the Bhargava-Kane-Lenstra-Poonen-Rains heuristics, which predict the distribution of Selmer groups of these abelian varieties. In joint work with Jordan Ellenberg, we prove a version of these heuristics: over function fields over the finite field $\mathbb{F}_q$, we show that the above heuristics are correct to within an error term in q, which goes to 0 as q grows. The main inputs are a new homological stability theorem in topology for a generalized version of Hurwitz spaces and an expression of average sizes of Selmer groups in terms of the number of rational points on these Hurwitz spaces over finite fields.

Rina Paucar (Universidad nacional de Ingenería (Uni)-Instituto de Matemáticas y Ciencias Afines (IMCA))

On a special case of Bloch-Beilinson conjectures

Abstract: In this short talk I will talk about a conjecture relating a distinguished element of the Chow group of zero cycles of a K3 surface over a number field, introduced by Beauville and Voisin, and rational points of this surface.

Chris Xu (UC San Diego)

Model-free quadratic Chabauty for modular curves

Abstract: In implementing quadratic Chabauty for modular curves, we describe how to potentially circumvent the need for a plane equation, which may be cumbersome to work with. In our context, differential forms become weight 2 cusp forms, and rational points become elliptic curves with prescribed level structure. We elaborate on a specific example: the nonsplit Cartan modular curve of level 27.

Session 2: Monday, July 8, 3:10 - 4:00

Monday	Speaker
3:10 pm	Ivan Aidun
3:17 pm	Lorenzo Furio
3:24 pm	Arav Karighattam
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3:38 pm	Boaz Moerman
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3:52 pm	Robin Visser

Ivan Aidun (UW-Madison)

Arithmetic Strength of Curves

Abstract: The strength of a homogeneous polynomial f is the least number of reducible forms $g_i h_i$ needed in an expression of the form $f = g_1 h_1 + g_2 h_2 + \ldots$. The collective strength of several homogeneous polynomials is the minimal strength of any homogeneous linear combination. These invariants have recently received great attention in the commutative algebra world in connection to progress on Stillman's conjecture. Ellenberg observed that over a non-algebraically closed field, the existence of a rational solution to $f = 0$ implies a certain kind of strength decomposition, suggesting that it may be profitable to ask "rational points" types of questions about the existence of strength decompositions, such as existence of decompositions locally, and when there is a Hasse-principle. In this talk, I will discuss the collective strength of two ternary quadrics over various fields of arithmetic interest.

Lorenzo Furio (Università di Pisa)

Recent progress on Serre's Uniformity Question

Abstract: In 1972 Serre proved his celebrated Open Image Theorem, stating that for any non-CM elliptic curve E defined over $\mathbb{Q}$ there exists a constant N such that the representation associated with the Galois action on the p-torsion points of E is surjective for $p > N$. In the same article, he asked whether the constant N can be taken to be independent of E, and this became known as Serre's Uniformity Question. In this talk, I will discuss the current progress towards an answer to this question, stating some classification results for the images of the p-adic representations and explicit bounds for the index of the image of the adelic representation.

Arav Karighattam (Harvard University)

Heegner Points on $y^2 = x^3 + p$

Abstract: In 1987, Satge proved via a Heegner point construction that every prime number congruent to 2 (mod 9), and the square of every prime number congruent to 5 (mod 9), is a sum of two cubes, by starting with a modular parametrization for the curve $y^2 = x^3 + 1$. By using a new modular parametrization in terms of division values of the Weierstrass p-function, I show that Satge's rational point is always twice a rational point. Using a quadratic twist modification to Satge's method, I construct nontrivial rational points on $y^2 = x^3 + p$ whenever p is a prime congruent to 5 (mod 36) and when the class number of the (non-Galois) cubic extension containing the cube root of p is odd. The proof of nontriviality requires expressing the fundamental unit and class number of this cubic extension in terms of the norm of a modular function of level 6, which is an analogue of a theorem of Dirichlet in 1840.

Elvira Lupoian (University College London)

Cuspidal Groups of Modular Jacobians

Abstract: The theorems of Manin and Drinfeld tell us the subgroup generated by the cusps of a congruence subgroup form a finite subgroup of the corresponding Jacobian. In a classical result, Ogg proved that for the modular curve $X_0(p)$, where $p \geq 5$ is prime, this group is a cyclic subgroup of order the numerator of $p - 1/12$. This subgroup is in fact the entire rational torsion subgroup of the Jacobian $J_0(p)$ - as proved by Mazur. In this short talk, we discuss the cuspidal subgroup for various covers of $X_0(p)$.

Boaz Moerman (Utrecht University)

Generalized Campana points and adelic approximation

Abstract: In recent years, there has been a lot of interest in the study of Campana points, which are special rational points related to squareful numbers. In this talk we will introduce a vast generalization of this notion, called M-points. For these points, we will study an analogue of strong approximation and characterize when this is satisfied on split toric varieties.

James Austin Myer (The CUNY Graduate Center)

(Toward) An Algorithm to (Explicitly) Produce a Regular Model of a Hyperelliptic Curve in (Bad) Characteristic (0, 2): A Criterion to Verify Regularity of the Normalization of a Candidate Model

Abstract: Given a hyperelliptic curve (defined over a "pleasant" field of characteristic 0 whose ring of integers is of mixed (bad) characteristic (0, 2)), we seek a regular model, i.e. a(n arithmetic) surface fibered over the (spectrum of the) ring of integers of the field whose generic fiber is the given curve, and with a special fiber: its avatar in characteristic 2. A strategy is afforded within a paper of Dino Lorenzini & Qing Liu: there exists a (regular) model of the projective line whose normalization in the function field of the given hyperelliptic curve is its sought after regular model. So, we seek such a regular model of the projective line... A candidate such model is gifted to us (explicitly) by work of Andrew Obus & Padmavathi Srinivasan. We establish a stepping stone across the river toward an algorithm to (explicitly) produce a regular model of any hyperelliptic curve in mixed (bad) characteristic (0, 2): a criterion to verify the regularity of the normalization of a candidate model of a hyperelliptic curve (equivalently, the normalization of the candidate model of the projective line of Obus & Srinivasan in the function field of the hyperelliptic curve).

Robin Visser (University of Warwick)

Computing genus 2 curves over $\mathbb{Q}$ whose Jacobian has good reduction away from 2

Abstract: A consequence of Faltings' celebrated proof of the Shafarevich conjecture is that there exist only finitely many principally polarised abelian surfaces over $\mathbb{Q}$ with good reduction away from 2. Whilst Smart computed a list of all 366 genus 2 curves over $\mathbb{Q}$ with good reduction away from 2, it remains an open problem, first posed by Poonen, to compute all such genus 2 curves whose Jacobians have good reduction away from 2. In this talk, I will present some work in progress on computing examples of such genus 2 curves. In particular, we can show there exist at least 512 genus 2 curves $C/\mathbb{Q}$ whose Jacobians have good reduction away from 2, with completeness guaranteed in cases where C has good reduction outside some small explicit sets of primes S

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Session 3: Tuesday, July 9, 11:15 - 12:05

Tuesday	Speaker
11:15 am	Elyes Boughattas
11:22 am	Jerry Yu Fu
11:29 am	Asimina Hamakiotes
11:36 am	Maleeha Khawaja
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Elyes Boughattas (University of Bath)
The tale of the Fibration Method
Abstract: Determining whether a given diophantine equation has a solution is a wide open question in number theory. For some varieties – e.g. quadrics – the existence of local points is enough to determine the existence of global points: this is known as the Hasse principle. Nevertheless, the latter does not hold for cubic forms, as shown by Selmer in 1951. Manin introduced in 1970 a set called the Brauer-Manin set, that is expected to describe all obstructions to the Hasse principle, at least for the wide family of rationally connected varieties. During this very short talk, I will say few words about the "fibration method" which predicts the behaviour of this Brauer-Manin setting in family.

Jerry Yu Fu (Caltech)
Isogenous elliptic curves in a family ordered by height
Abstract: Given a family of algebraic varieties, a natural question to ask is what type of properties of the generic fiber, and how those properties extend to other fibers. Let's explore this topic from an arithmetic point of view by looking at the scenario: Suppose we have a 1-dimensional family of pairs of elliptic curves over a number field K, with the generic fiber of this family being a pair of non-isogenous elliptic curves. Furthermore, suppose the (projective) height of the parametrizer is less than or equal to B. One may ask how does the property of "being isogenous" extends to the special fibers. Can we give a quantitative estimation for the number of specializations of height at most B, such that the two elliptic curves at the specializations are isogenous?

Asimina Hamakiotes (University of Connecticut)

Towards a classification of p^2-discriminant ideal twins over number fields

Abstract: Isogenous elliptic curves have the same conductor but not necessarily the same minimal discriminant ideal. In this project, we explicitly classify all p^2-isogenous elliptic curves defined over a number field with the same minimal discriminant ideal for odd prime p where $X_0(p^2)$ has genus 0, i.e., $p = 3$ or 5. As a consequence, we give a list of all p^2-isogenous discriminant (ideal) twins over $\mathbb{Q}$ for such p. This is joint work with Aly Deines, Andreea Iorga, Changningphaabi Namoijam, Manami Roy, and Lori Watson.

Maleeha Khawaja (University of Bristol)

Primitive algebraic points on curves

Abstract: The question of when a curve has only finitely many points of low degree has been studied intensively over the last few decades. We consider this question whilst also focusing on the Galois groups of these points, and in particular pay close attention to points with primitive Galois group. This talk is based on joint work with Samir Siksek.

Pietro Mercuri (Sapienza Università di Roma)

Arakelov canonical divisor and Bogomolov conjecture for modular curves

Abstract: We sketch how to describe the asymptotic behaviour of the self-intersection of the Arakelov canonical divisor for the classical Borel family of modular curves and we explain how from this one can deduce an effective version of Bogomolov conjecture for this family of curves.

Thibaut Misme (Trinity College Dublin (TCD))

Automatized computation of Odd Theta Caracteristic and 2-descent

Abstract: In this short talk I will briefly present the algorithm I am working on, aiming to compute the Galois action on the set of the odd theta characteristics or on the 2-torsion of any curve in an automatized way. This would ease any attempt of 2-descent on non-hyperelliptic curves, as it is the first necessary step to complete for one wishing to do so.

Madhavan Venkatesh (Indian Institute of Technology-Kanpur)

Effective bounds for Gabber's theorem

Abstract: Gabber's theorem concerns the torsion-freeness of the integral ℓ-adic cohomology of smooth projective varieties over finite fields for all but finitely many ell. In this talk, we give effective bounds for ell in terms of the degree of the variety in question with regard to the above theorem. The key ingredients are an effective version of Deligne's gcd theorem and mod-ell monodromy arguments.

Session 4: Tuesday, July 9, 3:10 - 4:00

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3:10 pm	Niven Achenjang
3:17 pm	Luca Ferrigno
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3:52 pm	Pratiksha Shingavekar

Niven Achenjang (MIT)

The average rank of elliptic curves is bounded, over any global field

Abstract: In the same paper that Mordell stated his famous conjecture, he also proved his theorem (later generalized by Weil) that the abelian group of $\mathbb{Q}$-points on an elliptic curve $E/\mathbb{Q}$ is finitely generated. This has led into a long and ongoing history of studying the possibilities and the "statistics" of the (free and torsion parts of the) groups $E(\mathbb{Q})$. In this lightning talk, I want to briefly survey a sliver of this history; namely, I will recap the question of computing the average rank of elliptic curves over a fixed global field K and highlight some of the progress on this question.

Luca Ferrigno (Università degli studi Roma Tre)

Isogeny relations in products of families of elliptic curves

Abstract: In this talk we will discuss a new result concerning the Zilber-Pink conjecture for a curve in the product of two fibered powers of elliptic schemes, provided that the curve satisfies a certain condition on the degrees of some of its coordinates.

Giorgio Navone (UCL)

Transcendental Brauer group of certain K3 surfaces

Abstract: The talk is about ongoing work computing the transcendental Brauer groups of a family of K3 surfaces, constructed from a planar cubic curve in a similar fashion to Kummer surfaces.

Benjamin York (University of Connecticut)

Models of CM elliptic curves with prescribed ℓ-adic Galois image

Abstract: In 2018, Lozano-Robledo provided a classification for ℓ-adic Galois representations attached to elliptic curves with complex multiplication (CM). In this talk, we will discuss a classification of Weierstrass models for CM elliptic curves with specified ℓ-adic Galois representation, and discuss our methods for proving this classification. This is joint work with Enrique Gonzalez-Jimenez and Alvaro Lozano-Robledo.

Roy Zhao (Caltech)

Algebraic Independence of Special Points on Shimura Varieties

Abstract: Given a correspondence V between a connected Shimura variety S and a commutative connected algebraic group T, we prove that the V-images of any n special points on S outside a proper Zariski closed subset are algebraically independent. One application proves multiplicative independence of differences of singular moduli, generalizing previous results by Pila-Tsimerman, and Aslanlyan-Eterović-Fowler. Another application proves that special points of S whose V-images lie in a finite-rank subgroup of T are contained in a finite union of proper special subvarieties of S, only dependent on the rank of the subgroup. This generalizes works of Pila-Tsimerman and Buium-Poonen. This work was done in collaboration with Yu Fu.

Sa'ar Zehavi (Ben Gurion University)

The Chabauty-Kim-Kantor method

Abstract: I will discuss the Chabauty-Kim-Kantor method, an extension of the Chabauty-Kim method that replaces the unipotent completions of the different realizations of the motivic fundamental group with their relative completions, as developed in Noam Kantor's PhD thesis. This method generalizes both the Lawrence-Venkatesh and Chabauty-Kim methods. I will present an overview of this approach, its motivations, and my recent unpublished results towards extending Kantor's program.

Pratiksha Shingavekar (Chennai Mathematical Institute)

A positive density of elliptic curves are diophantine stable in certain Galois extensions

Abstract: Let E be an elliptic curve over a number field K and L be a finite extension of K. Then E is said to be *diophantine stable* in L if the natural inclusion induces an equality $E(K) = E(L)$. We study the question of diophantine stability for a cyclic p-extension $L/\mathbb{Q}$, where $p \in \{3, 5\}$. In my recent a joint work with Anwesh Ray, we show that there exists an effective positive density of elliptic curves E defined over $\mathbb{Q}$, ordered by height, that are diophantine stable in L.