

Automatized computation of Odd Theta Characteristic and 2-descent

Thibaut Misme

The Mordell Conjecture: 100 years later
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We want to compute $Sel^{(2)}(\mathbb{Q})$ because

$$\text{rank}_{\mathbb{Q}}(J) \leq |Sel^{(2)}(\mathbb{Q})|$$

but

$$Sel^{(2)}(\mathbb{Q}) \subset H^1(\mathbb{Q}, J[2])$$

Problem

Problem: $H^1(\mathbb{Q}, J[2])$ NOT explicit
 \rightsquigarrow we look for $H^1(\mathbb{Q}, J[2]) \rightarrow H^1(\mathbb{Q}, G)$

We need:

- G a group with a Galois action (1)
- $H^1(\mathbb{Q}, G)$ explicit (2)
- $J[2] \hookrightarrow G$ Galois stable (3)

Solution

Solution: $G = \mathbb{F}_2^X$ with X a set with a known and explicit Galois action

\rightsquigarrow group (1) of explicit Cohomology (2)

We need to adjust X so that $J[2] \hookrightarrow G$

Example:

- (hyper)elliptic curves:
 $X = \{ \text{Weierstrass Points} \}$
- Generic curve:
 $X = J[2]$
or (better)
 $X = \{ \text{Odd Theta Characteristics} \}$