On A special case of Bloch-Beilinson conjectures

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The Mordell conjecture 100 years later

July 8, 2024

Constant cycles curves(ccc)

The notion of constant cycle curves originated when Beauville and Voisin in [BV04] described a distinguished element in the Chow group of 0-cycles of a K_3 surface.

More precisely, let S be a $K3$ surface over $\mathbb{C}$ or $\overline{\mathbb{Q}}$, they proved that there exists an element $c_S \in \mathsf{CH}_0(S)$ such that

$$\text{If } x \in C \subset S \text{ with } C \text{ a rational curve, then } [x] = c_S.$$

This property inspired the definition of a pointwise constant cycle curve introduced by D. Huybrechts in [Huy14]:

Definition (Pointwise constant cycle curve (pccc))

Let S be a projective K3 surface over a field k. A curve C in S is a *pointwise constant cycle curve (pccc)* if all closed points $x \in C$ define the same class $[x]$ in $\mathrm{CH}_0(S)$.

If k is algebraically closed, the above definition is equivalent to the following

Definition (Pointwise constant cycle curve (pccc))

Let S be a projective K3 surface over a field k. A curve C in S is a *pointwise constant cycle curve (pccc)* if all closed points $x \in C$ we have $[x] = c_S$ in $\mathrm{CH}_0(S)$.

When the ground field is algebraically closed, a constant cycle curve (ccc) is a pccc.

The Conjecture

conjecture (Special case of Bloch-Beilinson conjectures)

If S is a K3 surface over $\overline{\mathbb{Q}}$ and if $x \in S(\overline{\mathbb{Q}})$ is a $\overline{\mathbb{Q}}$-rational point, then $[x] = c_S$ ([Huy14, §2]).

Ideas how to prove it

- Bogomolov pointed out that a positive answer is possible for the following question

  Question: Could it be possible that any $\overline{\mathbb{Q}}$-rational point lies in a rational curve?

  Note that a positive answer to this question would prove the conjecture. Because if any $\overline{\mathbb{Q}}$-rational point x of S lies in a rational curve, then by the result proved by Beauville and Voisin we have that $[x] = c_S$.
- On the other hand, D. Huybrechts pointed out that a positive answer is possible for the above question if we replace rational curves for ccc. Note that a positive answer to this last question would also prove the conjecture.

References I

[BV04] Arnaud Beauville and Claire Voisin. "*On the Chow ring of a K3 surface*". In: *Journal of Algebraic Geometry* 13.3 (2004), pp. 417–426.

[Huy14] Daniel Huybrechts. "*Curves and cycles on K3 surfaces*". In: *Foundation Compositio Mathematica* (2014).