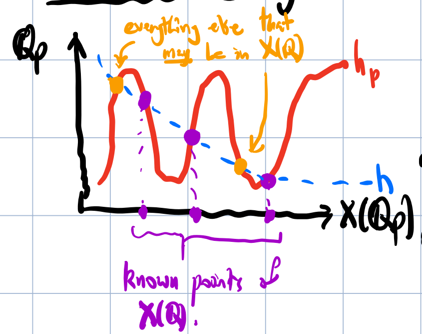
Model-free quadratic Chabauty for modular curves. W.I.P. (w/ Y. Huang, J. Lau)

① Quadratic Chabauty.



- Let h (resp. h_p) denote the global (resp. local) height functions on X. - fixed Z, and b.
- If $x \in X(\mathbb{Q})$, then $h_p(x) = h(x)$. — assuming pot. good reduction
- Compute h_p directly, and compute h by "quadratic interpolation". Then find $Z(h_p - h)$.

Can we do this without finding a good model for X?

② Computing h_p.

Need representatives of $H^1_{dR}(X)$ (w.h. m.f.), not just $H^0(X, \Omega^1)$ (cusp forms)

Idea: Construct modular forms with all zeros and poles on CM points/cusps, ~~and divide them to hopefully get w.h.m.f.~~ "Borcherds products".

③ Computing h.

We cannot "quadratically interpolate" if $X(\mathbb{Q})$ isn't large enough.

Idea: Compute h at CM divisors via p-adic Gross-Zagier formula, and then obtain h via another G-Z formula. (Hashimoto '22 for $X_0(N)$.)

Thank you for listening!